Appendix

At the beginning of each period t, $t = 1, \cdots, T$, the utility has x_t as its REC level and u_t as its cumulative demand. The utility also observes the buying and selling prices of unbundled RECs $R_t = (b_t, s_t)$, as well as the prices of REC-bundled energy and regular energy in the forward market $P_t = (p_{1t}, p_{2t})$. The utility makes decisions in two stages. In stage one, it trades in the REC market, adjusting its REC level to $\bar{x}_t$. In stage two, it purchases electricity from the forward market: y_{1t} of renewable energy and $y_{2t} - y_{1t}$ of regular energy. We write two cost-to-go functions for these two stages.

In stage one, the decision variable is $\bar{x}_t$, we write the cost-to-go function as

$$V_t(x_t, u_t, R_t, P_t) = \min_{\bar{x}_t} \{C_t(\bar{x}_t - x_t, R_t) + W_t(\bar{x}_t, u_t, R_t, P_t)\}, \tag{1}$$

where $C_t(\bar{x}_t - x_t, R_t) = b_t(\bar{x}_t - x_t)^+ - s_t(x_t - \bar{x}_t)^+$ is the cost or revenue when the utility purchases or sells RECs in the REC market, $W_t(\bar{x}_t, u_t, R_t, P_t)$ is the cost-to-go function for stage two.

In stage two, the decision variables are y_{1t} and y_{2t}, we write the cost-to-go function as

$$\begin{aligned} W_t(\bar{x}_t, u_t, R_t, P_t) = \min_{\substack{y_{1t} \geq 0 \\ y_{2t} \geq y_{1t}}} \{&p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t}) + G_t(y_{2t}) \\ &+ \gamma E_t[V_{t+1}(\bar{x}_t + y_{1t}, u_t + D_t, R_{t+1}, P_{t+1})]\}. \end{aligned} \tag{2}$$

$G_t(y_{2t})$ is the utility's expectation of its expense or revenue in the spot market, with a positive value implying expense and a negative value implying revenue. We do not restrict the specific form of $G_t(y_{2t})$, rather assume four properties.

(1) $G_t(y_{2t})$ is decreasing in y_{2t}. The more electricity the utility has purchased from the forward market, the less (more) it needs to purchase from (sell to) the spot market, hence the less the cost (the more the revenue).

(2) $G_t(y_{2t})$ is differentiable, and $|dG_t(y_{2t})/dy_{2t}| > p_{2t}$ when $G_t(y_{2t}) \geq 0$, and $|dG_t(y_{2t})/dy_{2t}| < p_{2t}$ when $G_t(y_{2t}) \leq 0$. When $G_t(y_{2t}) \geq 0$, it represents the cost from purchasing additional energy from the spot market. We assume the marginal cost of purchasing energy from the spot market is more than the unit cost of purchasing regular energy from the forward market. Hence the utility strictly prefers purchasing from the forward market. When $G_t(y_{2t}) \leq 0$, it represents the revenue from selling excessive energy to the spot market. We assume the marginal revenue of selling energy to the spot market is less than the unit price of regular energy in the forward market. Hence the utility cannot make a profit by over-purchasing energy from the forward market and then selling into the spot market.

(3) $G_t(y_{2t})$ is convex in y_{2t}. We assume the marginal revenue of selling energy in the spot market declines as the selling amount increases, and the marginal cost of purchasing electricity from the spot market increases as the purchasing amount increases.

(4) $dG_t(y_{2t})/dy_{2t} \to 0$ as $y_{2t} \to +\infty$. The marginal revenue of selling energy to the spot market approaches zero as the selling amount approaches infinity.

Lemma 1. *$W_t(x_t,\, u_t,\, R_t,\, P_t)$ and $V_t(x_t,\, u_t,\, R_t,\, P_t)$ are submodular on (x_t, u_t).*

Proof of Lemma 1:
We prove this lemma by induction in three steps:

(i) $V_{T+1}(x_{T+1}, u_{T+1})$ is submodular on (x_{T+1}, u_{T+1});
(ii) If $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ is submodular on (x_{t+1}, u_{t+1}), then $W_t(\bar{x}_t, u_t, R_t, P_t)$ is submodular on $(\bar{x}_t, u_t)$;
(iii) If $W_t(\bar{x}_t, u_t, R_t, P_t)$ is submodular on $(\bar{x}_t, u_t)$, then $V_t(x_t, u_t, R_t, P_t)$ is submodular on (x_t, u_t).

We show the proofs of these three steps in the following.

(i) Since πx_{T+1}^+ is convex in x_{T+1}, $V_{T+1}(x_{T+1}, u_{T+1}) = \pi(\alpha u_{T+1} - x_{T+1})^+$ is submodular on (x_{T+1}, u_{T+1}) (Topkis (1998), Lemma 2.6.2).
(ii) According to equation (2), by the preservation of submodularity (Topkis (1998), Theorem 2.7.6), it is sufficient to show:
  (a) $\{((\bar{x}_t, u_t), (y_{1t}, y_{2t})) : u_t \geq 0, y_{1t} \geq 0, y_{2t} \geq y_{1t}\}$ forms a lattice; and
  (b) $p_{1t}y_{1t}+p_{2t}(y_{2t}-y_{1t})+G_t(y_{2t})+\gamma E_t[V_{t+1}(\bar{x}_t+y_{1t},\, u_t+D_t, R_{t+1}, P_{t+1})]$ is submodular on $((\bar{x}_t, u_t), (y_{1t}, y_{2t}))$.

  (a) is easy to verify. Now we prove (b). Note $p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t})$ is linear in (y_{1t}, y_{2t}) and $G_t(y_{2t})$ is a function of a single variable, thus, $p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t}) + G_t(y_{2t})$ is submodular on (y_{1t}, y_{2t}). If we can show $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is submodular on $((\bar{x}_t, u_t), y_{1t})$, then (b) is true because the sum of two submodular functions is also submodular. We know that $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is submodular on $(\bar{x}_t, u_t)$ and (y_{1t}, u_t) by the assumption. Also because $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ is decreasing in x_{t+1}, $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is submodular on $(\bar{x}_t, y_{1t})$. Therefore, $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is submodular on $((\bar{x}_t, u_t), y_{1t})$. Thus we have proved (b). Therefore, we have proved (ii) .
(iii) Since $b_t x_t^+ - s_t(-x_t)^+$ is convex in x_t, $C_t(\bar{x}_t - x_t, R_t) = b_t(\bar{x}_t - x_t)^+ - s_t(x_t - \bar{x}_t)^+$ is submodular on $(\bar{x}_t, x_t)$ (Topkis (1998), Lemma 2.6.2). Because $W_t(\bar{x}_t, u_t, R_t, P_t)$ is submodular on $(\bar{x}_t, u_t)$ by assumption, $C_t(\bar{x}_t - x_t, R_t) + W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t)$ is submodular on $(x_t, u_t, \bar{x}_t)$. By the preservation of submodularity (Topkis (1998), Theorem 2.7.6), $V_t(x_t, u_t, R_t, P_t)$ is submodular on (x_t, u_t).

Lemma 2. *$W_t(x_t,\, u_t,\, R_t,\, P_t)$ and $V_t(x_t,\, u_t,\, R_t,\, P_t)$ are jointly convex on $(x_t,\, u_t)$.*

Proof of Lemma 2:
We prove this lemma by induction in three steps:

(i) $V_{T+1}(x_{T+1},\, u_{T+1}) = \pi(\alpha u_{T+1} - x_{T+1})^+$ is jointly convex on $(x_{T+1},\, u_{T+1})$;
(ii) If $V_{t+1}(x_{t+1},\, u_{t+1},\, R_{t+1}, P_{t+1})$ is jointly convex on $(x_{t+1},\, u_{t+1})$, then $W_t(\bar{x}_t,\, u_t,\, R_t, P_t)$ is jointly convex on $(\bar{x}_t,\, u_t)$;

(iii) If $W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t)$ is jointly convex on $(\bar{x}_t,\, u_t)$, then $V_t(x_t,\, u_t,\, R_t,\, P_t)$ is jointly convex on $(x_t,\, u_t)$.

We show the proofs of these three steps in the following.

(i) Since πx_{T+1}^{+} is convex in x_{T+1}, $V_{T+1}(x_{T+1},\, u_{T+1}) = \pi(\alpha u_{T+1} - x_{T+1})^{+} = \pi((\alpha, -1)(u_{T+1}, x_{T+1})^{\top})^{+}$ is jointly convex on $(x_{T+1},\, u_{T+1})$ by the preservation of convexity under composition with an affine mapping (Boyd and Vandenberghe, 2004).

(ii) According to equation (2), by the preservation of convexity under minimization, it is sufficient to show:

(a) $\{(y_{1t}, y_{2t}) : y_{1t} \geq 0, y_{2t} \geq y_{1t}\}$ is a convex set; and

(b) $p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t}) + G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]$ is convex on $(\bar{x}_t, u_t, y_{1t}, y_{2t})$.

(a) is easy to verify. Now we prove (b). Since $V_{t+1}(x_{t+1},\, u_{t+1},\, R_{t+1}, P_{t+1})$ is jointly convex on $(x_{t+1},\, u_{t+1})$, $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is jointly convex in $(\bar{x}_t, u_t)$ and (y_{1t}, u_t), also jointly convex in $(\bar{x}_t, y_{1t})$ by the preservation of convexity under composition with an affine mapping. Thus $V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, R_{t+1}, P_{t+1})$ is convex in $(\bar{x}_t, u_t, y_{1t})$. Then $E_t[V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]$ is convex in $(\bar{x}_t, u_t, y_{1t})$ by preservation of convexity under nonnegative weighted sums (Boyd and Vandenberghe, 2004). In addition, $G_t(y_{2t})$ is convex in y_{2t}. Thus, we have proved (b). Therefor, we have proved (ii).

(iii) According to equation (1), by the preservation of convexity under minimization, it is sufficient to show $C_t(\bar{x}_t - x_t,\, R_t) + W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t)$ is convex in $(x_t, u_t, \bar{x}_t)$. Since $b_t x_t^{+} - s_t(-x_t)^{+}$ is convex in x_t, by the preservation of convexity under composition with an affine mapping, $C_t(\bar{x}_t - x_t, R_t) = b(\bar{x}_t - x_t)^{+} - s_t(x_t - \bar{x}_t)^{+} = b_t((1, -1)(\bar{x}_t, x_t)^{\top})^{+} - s_t(-(1, -1)(\bar{x}_t, x_t)^{\top})^{+}$ is jointly convex on $(\bar{x}_t, x_t)$. In addition, $W(\bar{x}_t,\, u_t,\, R_t,\, P_t)$ is convex in $(\bar{x}_t, u_t)$ by assumption. Thus $C_t(\bar{x}_t - x_t,\, R_t) + W(\bar{x}_t,\, u_{,t}\, R_t,\, P_t)$ is convex in $(x_t, u_t, \bar{x}_t)$.

Proposition 1. *In each period t, $t = 1, \ldots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$, the utility's optimal REC trading policy is a target intervel policy with two thresholds $L_t(u_t,\, R_t,\, P_t)$ and $H_t(u_t,\, R_t,\, P_t)$, with $L_t(u_t,\, R_t,\, P_t) \leq H_t(u_t,\, R_t,\, P_t)$. The optimal REC level after REC trading is*

$$\bar{x}_t^{*} = \begin{cases} L_t(u_t,\, R_t,\, P_t) & \text{if } x_t \leq L_t(u_t,\, R_t,\, P_t); \\ x_t & \text{if } L_t(u_t,\, R_t,\, P_t) < x_t < H_t(u_t,\, R_t,\, P_t); \\ H_t(u_t,\, R_t,\, P_t) & \text{if } x_t \geq H_t(u_t,\, R_t,\, P_t). \end{cases}$$

Proof of Proposition 1:

We can write equation (1) as

$$
\begin{aligned}
V_t(x_t, u_t, R_t, P_t) &= \min_{\bar{x}_t}\{C_t(\bar{x}_t - x_t, R_t) + W_t(\bar{x}_t, u_t, R_t, P_t)\} \\
&= \min_{\bar{x}_t}\{\min_{\bar{x}_t \geq x_t}\{b_t\bar{x}_t - b_t x_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}, \\
&\qquad \min_{\bar{x}_t \leq x_t}\{-s_t x_t + s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}\} \\
&= \min_{\bar{x}_t}\{-b_t x_t + \min_{\bar{x}_t \geq x_t}\{b_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}, \\
&\qquad -s_t x_t + \min_{\bar{x}_t \leq x_t}\{s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}\}.
\end{aligned} \tag{3}
$$

Define

$$
\begin{aligned}
L_t(u_t, R_t, P_t) &= \arg\min_{\bar{x}_t}\{b_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}, \\
H_t(u_t, R_t, P_t) &= \arg\min_{\bar{x}_t}\{s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}.
\end{aligned}
$$

Since $b_t\bar{x}_t(s_t\bar{x}_t)$ is submodular, and $W_t(\bar{x}_t, u_t, R_t, P_t)$ is submodular on $(\bar{x}_t, u_t)$ (Lemma 1), so $L_t(u_t, R_t, P_t)$ and $H_t(u_t, R_t, P_t)$ increases in u_t. Since $W_t(\bar{x}_t, u_t, R_t, P_t)$ is convex in $\bar{x}_t$ (Lemma 2) and $b_t \geq s_t$, we have $L_t(u_t, R_t, P_t) \leq H_t(u_t, R_t, P_t)$. Consider the two sub-optimization problems in (3). We refer to $-b_t x_t + \min_{\bar{x}_t \geq x_t}\{b_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}$ as optimization problem I and $-s_t x_t + \min_{\bar{x}_t \leq x_t}\{s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)\}$ as optimization problem II.

(a) If $x_t \geq H_t(u_t, R_t, P_t)$, then $x_t \geq L_t(u_t, R_t, P_t)$. Let us consider optimization problem I first. From the definition of $L_t(u_t, R_t, P_t)$ and convexity of $W_t(\bar{x}_t, u_t, R_t, P_t)$ on $\bar{x}_t$, $b_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)$ is increasing in $\bar{x}_t$ when $\bar{x}_t \geq x_t$. Thus, for optimization problem I, the optimal solution is $\bar{x}_t = x_t$ which gives an optimal value $-b_t x_t + b_t x_t + W_t(x_t, u_t, R_t, P_t) = W_t(x_t, u_t, R_t, P_t)$. Now Let us consider optimization problem II. Note $\bar{x}_t = x_t$ is a feasible solution for optimization problem II. From the definition of $H_t(u_t, R_t, P_t)$, we have $s_t H_t(u_t, R_t, P_t) + W_t(H_t(u_t, R_t, P_t), u_t, R_t, P_t) \leq s_t x_t + W_t(x_t, u_t, R_t, P_t)$. Therefore,
$-s_t x_t + s_t H_t(u_t, R_t, P_t) + W_t(H_t(u_t, R_t, P_t), u_t, R_t, P_t) \leq W_t(x_t, u_t, R_t, P_t)$, meaning that the optimal value of optimization problem II is less than the optimal value of optimization problem I. Thus, for optimization problem (3), $\bar{x}_t^* = H_t(u_t, R_t, P_t)$.

(b) If $x_t \leq L_t(u_t, R_t, P_t)$, then $x_t \leq H_t(u_t, R_t, P_t)$. Let us consider optimization problem II first. From the definition of $H_t(u_t, R_t, P_t)$ and convexity of $W_t(\bar{x}_t, u_t, R_t, P_t)$ on $\bar{x}_t$, $s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)$ is decreasing in $\bar{x}_t$ when $\bar{x}_t \leq x_t$. Thus, for optimization problem II, the optimal solution is $\bar{x}_t = x_t$ which gives an optimal value $-s_t x_t + s_t x_t + W_t(x_t, u_t, R_t, Pv) = W_t(x_t, u_t, R_t, P_t)$. Now let us consider optimization problem I. Note $\bar{x}_t = x_t$ is a feasible solution for optimization problem I. From the definition of $L_t(u_t, R_t, P_t)$, we have $b_t L_t(u_t, R_t, P_t) + W_t(L_t(u_t, R_t, P_t), u_t, R_t, P_t) \leq b_t x_t + W_t(x_t, u_t, R_t, P_t)$. Therefore,
$-b_t x_t + b_t L_t(u_t, R_t, P_t) + W_t(L_t(u_t, R_t, P_t), u_t, R_t, P_t) \leq W_t(x_t, u_t, R_t, P_t)$,

meaning that the optimal value of optimization problem I is less than the optimal value of optimization problem II. Thus, for optimization problem (3), $\bar{x}_t^* = L_t(u_t, R_t, P_t)$.

(c) If $L_t(u_t, R_t, P_t) \le x_t \le H_t(u_t, R_t, P_t)$, $b_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)$ is increasing in $\bar{x}_t$ when $\bar{x}_t \ge x_t$, $s_t\bar{x}_t + W_t(\bar{x}_t, u_t, R_t, P_t)$ is decreasing in $\bar{x}_t$ when $\bar{x}_t \le x_t$. Thus, for both optimization problem I and II, the optimal solution is $\bar{x}_t^* = x_t$. Therefore, for optimization problem (3), $\bar{x}_t^* = x_t$.

Theorem 1. *In each period t, $t = 1, \ldots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$,*

(a) *If $\triangle_t \ge b_t$, it is optimal for the utility to purchase only regular energy, $y_{1t}^* = 0$, $y_{2t}^* = \arg\min_{y_{2t}\ge 0}\{p_{2t}y_{2t} + G_t(y_{2t})\} \triangleq S_{2t}(p_{2t})$;*

(b) *If $\triangle_t \le s_t$, it is optimal for the utility to purchase only REC-bundled energy,*

$$y_{1t}^* = y_{2t}^* = \begin{cases} S_{1t}^L(p_{1t}, b_t) & \text{if } x_t \le L_t(u_t,\, R_t,\, P_t), \\ s_{1t}(x_t,\, u_t,\, R_t,\, P_t) & \text{if } L_t(u_t,\, R_t,\, P_t) < x_t < H_t(u_t,\, R_t,\, P_t), \\ S_{1t}^H(p_{1t}, s_t) & \text{if } x_t \ge H_t(u_t,\, R_t,\, P_t). \end{cases}$$

where

$$\begin{aligned} S_{1t}^L(p_{1t}, b_t) &= \arg\min_{y_{1t}\ge 0}\{(p_{1t} - b_t)y_{1t} + G_t(y_{1t})\}, \\ s_{1t}(x_t,\, u_t,\, R_t,\, P_t) &= \arg\min_{y_{1t}\ge 0}\{p_{1t}y_{1t} + G_t(y_{1t}) \\ &\qquad + \gamma E[V_{t+1}(x_t + y_{1t},\, u_t + D_t,\, R_{t+1},\, P_{t+1})]\}, \\ S_{1t}^H(p_{1t}, s_t) &= \arg\min_{y_{1t}\ge 0}\{(p_{1t} - s_t)y_{1t} + G_t(y_{1t})\}. \end{aligned}$$

Proof of Theorem 1:

(a) In period t, if $\triangle_t \ge b_t$, we want to show that it is optimal to purchase only regular energy. Suppose the optimal action involves purchasing REC-bundled renewable energy. Note $\triangle_t \ge b_t$ implies $p_{1t} \ge p_{2t} + b_t$. This means for each unit of REC-bundled energy, the utility can get the equivalent product by combining one unit of regular energy and one REC but at a lower price. Therefore, if $\triangle_t \ge b_t$ it is always better to purchase only regular energy, i.e., $y_{1t}^* = 0$. We can write the cost-to-go function after the REC trading (2) as

$$\begin{aligned} &W_t(\bar{x}_t, u_t, R_t, P_t) \\ &= \min_{y_{2t}\ge 0}\{p_{2t}y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}. \end{aligned}$$

Thus

$$y_{2t}^* = \arg\min_{y_{2t}\ge 0}\{p_{2t}y_{2t} + G_t(y_{2t})\} = S_{2t}(p_{2t}).$$

The convexity of $G_t(y_{2t})$ implies that $S_{2t}(p_{2t})$ is decreasing in p_{2t}.

(b) In period t, if $\triangle_t \leq s_t$, we want to show that it is optimal to purchase only REC-bundled energy. Suppose the optimal action involves purchasing regular energy. Note that $\triangle_t \leq s_t$ implies $p_{2t} \geq p_{1t} - s_t$. This means for each unit of regular energy, the utility can get the equivalent product by purchasing one unit of REC-bundled energy and then selling the REC comes with it. The resulting price is lower than purchasing regular energy directly. Therefore, if $\triangle_t \leq s_t$ it is always better to purchase only REC-bundled energy, i.e., $y_{1t}^* = y_{2t}^*$. We consider three cases based on the utility's REC level at the beginning of period t. For ease of analysis we write $w_t = \bar{x}_t + y_{1t}$ as the REC level at the end of period t.

(i) When $x_t \leq L_t(u_t, R_t, P_t)$, by Proposition 1 it is optimal for the utility to purchase RECs to increase its REC level up to $\bar{x}_t^* = L_t(u_t, R_t, P_t)$. Therefore, we can write the cost-to-go function as

$$\begin{aligned} V_t(x_t, u_t, R_t, P_t) = &\min_{\substack{y_{1t} \geq 0 \\ w_t}} \{b_t[(w_t - y_{1t}) - x_t)] + p_{1t}y_{1t} + G_t(y_{1t}) \\ &+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\} \\ = &- b_t x_t + \min_{\substack{y_{1t} \geq 0 \\ w_t}} \{(p_{1t} - b_t)y_{1t} + G_t(y_{1t}) + b_t w_t \\ &+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}. \end{aligned}$$

Note the objective function in the bracket is composed of a convex function on y_{1t} and a convex function on w_t, thus

$$w_t^* = \arg\min_{w_t} \{b_t w_t + \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\} \triangleq w_t^L(u_t, R_t, P_t),$$

$$y_{1t}^* = \arg\min_{y_{1t} \geq 0} \{(p_{1t} - b_t)y_{1t} + G_t(y_{1t})\} = S_{1t}^L(p_1, b_t),$$

where $w_t^L(u_t, R_t, P_t)$ is the utility's optimal REC level at the end of period t when it purchases unbundled RECs. We can write

$$L_t(u_t, R_t, P_t) = \bar{x}_t^* = w_t^* - y_{1t}^* = w_t^L(u_t, R_t, P_t) - S_{1t}^L(p_1, b_t).$$

(ii) When $x_t \geq H_t(u_t, R_t, P_t)$, by Proposition 1 it is optimal for the utility to sell RECs to decrease its REC level to $\bar{x}_t^* = H_t(u_t, R_t, P_t)$. Therefore, we can write the cost-to-go function as

$$\begin{aligned} V_t(x_t, u_t, R_t, P_t) = &\min_{\substack{y_{1t} \geq 0 \\ w_t}} \{-s_t[x_t - (w_t - y_{1t})] + p_{1t}y_{1t} + G_t(y_{1t}) \\ &+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\} \\ = &- s_t x_t + \min_{\substack{y_{1t} \geq 0 \\ w_t}} \{(p_{1t} - s_t)y_{1t} + G_t(y_{1t}) + s_t w_t \\ &+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}. \end{aligned}$$

Thus

$$w_t^* = \arg\min_{w_t} \{s_t w_t + \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\} \triangleq w_t^H(u_t, R_t, P_t),$$

$$y_{1t}^* = \arg\min_{y_{1t} \geq 0} \{(p_{1t} - s_t)y_{1t} + G_t(y_{1t})\} = S_{1t}^H(p_{1t}, s_t),$$

where $w_t^H(u_t, R_t, P_t)$ is the utility's optimal REC level at the end of period t when it sells unbundled RECs. We can write

$$H_t(u_t, R_t, P_t) = \bar{x}_t^* = w_t^* - y_{1t}^* = w_t^H(u_t, R_t, P_t) - S_{1t}^H(p_{1t}, s_t).$$

(iii) When $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$, by Proposition 1 it is optimal for the utility not to trade RECs, i.e., $\bar{x}_t^* = x_t$. Therefore, we can write the cost-to-go function as

$$\begin{aligned} &V_t(x_t, u_t, R_t, P_t) \\ =&\min_{y_{1t}\geq 0}\{p_{1t}y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(x_t + y_{1t}, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\}. \end{aligned}$$

Thus

$$\begin{aligned} y_{1t}^* =&\arg\min_{y_{1t}\geq 0}\{p_{1t}y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(x_t + y_{1t}, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\ =&s_{1t}(x_t, u_t, R_t, P_t). \end{aligned}$$

We can show that $S_{1t}^L(p_{1t}, b_t)$ is strictly positive. In order to show $S_{1t}^L(p_{1t}, b_t)$ is positive, it is sufficient to show that $(p_{1t}-b_t)y_{1t}+G_t(y_{1t})$ is strictly decreasing in y_{1t} at $y_{1t} = 0$, and thus it is sufficient to show that $p_{1t} - b_t + dG_t(y_{1t})/dy_{1t} < 0$ at $y_{1t} = 0$. Note that $G_t(0)$ stands for the utility's expected cost in the spot market when it has not purchased any energy from the forward market. In this case the utility will purchase from the spot market, thus $G_t(0) > 0$. The second property of $G_t(y_{1t})$ states that $|dG_t(y_{1t})/dy_{1t}| > p_{2t}$ when $G_t(y_{1t}) \geq 0$. Therefore, at $y_{1t} = 0$, $dG_t(y_{1t})/dy_{1t} < -p_{2t}$, thus $p_{1t} - b_t + dG_t(y_{1t})/dy_{1t} < p_{1t} - b_t - p_{2t} = \triangle_t - b_t$. Because in Theorem 1, the condition is $\triangle_t < s_t$, and because $s_t < b_t$, thus $\triangle_t < b_t$. Therefore, $p_{1t} - b_t + dG_t(y_{1t})/dy_{1t} < 0$ at $y_{1t} = 0$. Thus $S_1^L(p_{1t}, b_t)$ is strictly positive. Similarly, we can show that $S_1^H(p_{1t}, s_t)$ is strictly positive.

We can show the following properties of these purchasing quantities.

(1) $S_{1t}^L(p_{1t}, b_t)$ is decreasing in p_{1t} and increasing in b_t.
Since $f(x, y) = xy$ is supermodular on (x, y), and $(p_{1t}-b_t)y_{1t}+G_t(y_{1t})$ is supermodular on (p_{1t}, y_{1t}) and submodular on (b_t, y_{1t}), we have $S_{1t}^L(p_{1t}, b_t)$ is decreasing in p_{1t} and increasing in b_t (Topkis (1998), Theorem 2.8.2).

(2) $S_{1t}^H(p_{1t}, s_t)$ is decreasing in p_{1t} and increasing in s_t. Proof is similar to (1).

(3) $s_{1t}(x_t,\ u_t,\ R_t, P_t)$ is decreasing in x_t and increasing in u_t.
Define a function $g(x_t, u_t, R_t, P_t, y_{1t})$ as

$$\begin{aligned} &s_{1t}(x_t, u_t, R_t, P_t) \\ =&\arg\min_{y_{1t}\geq 0}\{p_{1t}y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(x_t + y_{1t}, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\ =&\arg\min_{y_{1t}\geq 0} g(x_t, u_t, R_t, P_t, y_{1t}). \end{aligned}$$

In order to show $s_{1t}(x_t, u_t, R_t, P_t)$ is decreasing in x_t, it is sufficient to show that $g(x_t, u_t, R_t, P_t, y_{1t})$ is supermodular on (x_t, y_{1t}) (Topkis

(1998), Theorem 2.8.2). Since $\{(x_t, y_{1t}) : y_{1t} \geq 0\}$ is a sublattice of $\mathbf{R}^2$, and $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ is convex on $x_{t+1} \in \mathbf{R}$, thus, $V_{t+1}(x_t + y_{1t}, u_t, R_t, P_t)$ is supermodular on (x_t, y_{1t}) (Topkis (1998), Lemma 2.6.2). Thus, $g(x_t, u_t, R_t, P_t, y_{1t})$ is supermodular on (x_t, y_{1t}). In order to show $s_{1t}(x_t, u_t, R_t, P_t)$ is increasing in u_t, it is sufficient to show that $g(x_t, u_t, R_t, P_t, y_{1t})$ is submodular on (u_t, y_{1t}), which is true since $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ is submodular on (x_{t+1}, u_{t+1}).

(4) $S^L_{1t}(p_{1t}, b_t) \geq s_{1t}(x_t,\ u_t,\ R_t, P_t) \geq S^H_{1t}(p_{1t}, s_t)$. This is straightforward from the convexity of $G_t(y_{1t})$ and the assumption $s_t \leq b_t$.

Lemma 3. *In period t, $t = 1, \cdots, T$, given state (x_t, u_t, R_t, P_t), any feasible action $(\bar{x}_t, y_{1t}, y_{2t})$ with $y_{2t} \geq y_{1t} \geq 0$ can be categorized into two types according to either $\bar{x}_t \leq x_t$ or $\bar{x}_t \geq x_t$:*

> *Type one: when $\bar{x}_t \leq x_t$, the utility sells RECs, we can characterize the action as $(-r, A, B)(r \geq 0, A \geq 0, B \geq 0)$ which means selling r units of RECs, buying A units of REC-bundled energy and B units of regular energy.*
>
> *Type two: when $\bar{x}_t \geq x_t$, the utility buys RECs, we can characterize the action as $(+r, A, B)(r \geq 0, A \geq 0, B \geq 0)$ which means buying r units of unbundled RECs, buying A units of REC-bundled energy and B units of regular energy.*

We make two observations as follows:

(a) *When $\triangle_t \geq b_t$, if the optimal action is type one $(-r, A, B)$, then $r \geq A$.*

(b) *When $\triangle_t \leq s_t$, if the optimal action is type two $(+r, A, B)$, then $r \geq B$.*

Proof of Lemma 3:

We prove this Lemma by contradiction.

(a) When $\triangle_t \geq b_t$, suppose $\mathbf{a} = (-r, A, B)$ is the optimal action and $r < A$. Consider another action $\mathbf{a}' = (0, A - r, B + r)$. By taking either action $\mathbf{a}$ or $\mathbf{a}'$, the utility obtains $A - r$ units of RECs and $A + B$ units of electricity in period t. Comparing the costs of these two actions, we have

$$\begin{aligned} cost(\mathbf{a}) - cost(\mathbf{a}') &= - s_t r + p_{1t}A + p_{2t}B - [p_{1t}(A - r) + p_{2t}(B + r)] \\ &= - s_t r + p_{1t} r - p_{2t} r \\ &= r(\triangle_t - s_t) \geq 0. \end{aligned}$$

Thus $\mathbf{a}'$ is a better action than $\mathbf{a}$, contradicts with the optimality of $\mathbf{a}$. Thus we have proved $r \geq A$.

(b) When $\triangle_t \leq s_t$, suppose $\mathbf{b} = (+r, A, B)$ is the optimal action and $r < B$. Consider another action $\mathbf{b}' = (0, A + r, B - r)$. By taking either action $\mathbf{b}$ or $\mathbf{b}'$, the utility obtains $(A + r)$ units of RECs and $(A + B)$ units of electricity in period t. Comparing the the costs of these two actions, we have

$$\begin{aligned} cost(\mathbf{b}) - cost(\mathbf{b}') &= b_t r + p_{1t}A + p_{2t}B - [p_{1t}(A + r) + p_{2t}(B - r)] \\ &= b_t r - p_{1t} r + p_{2t} r \\ &= r(b_t - \triangle_t) \geq 0. \end{aligned}$$

Thus $\mathbf{b}'$ is a better action than $\mathbf{b}$, contradicts with the optimality of $\mathbf{b}$. Thus we have proved $r \geq B$.

Theorem 2. *In each period $t = 1, \dots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$, if $s_t < \triangle_t < b_t$, then*

(a) *When $x_t \leq L_t(u_t,\, R_t,\, P_t)$, it is optimal for the utility to purchase only REC-bundled energy, and $y_{1t}^* = y_{2t}^* = S_{1t}^L(p_{1t}, b_t)$;*

(b) *When $x_t \geq H_t(u_t,\, R_t,\, P_t)$, it is optimal for the utility to purchase only regular energy, and $y_{1t}^* = 0$, $y_{2t}^* = S_{2t}(p_{2t})$.*

Proof of Theorem 2:
We prove this proposition by sample path and contradiction. We consider the case where $s_t < \triangle_t < b_t$.

(a) When $x_t \leq L_t(u_t, R_t, P_t)$, according to Proposition 1 it is optimal to purchase RECs. Therefore, we can write the optimal action as type two $\mathbf{b} = (+r, A, B)$. By Lemma 3 we know that $r \geq B$. Consider another action $\mathbf{b}'' = (r - B, A + B, 0)$. By taking either of action $\mathbf{b}$ or $\mathbf{b}''$, the utility gains $r + A$ units of RECs and $A + B$ units of electricity in period t. Compare the costs of these two actions, we have

$$\begin{aligned} cost(\mathbf{b}) - cost(\mathbf{b}'') &= b_t r + p_{1t}A + p_{2t}B - [b_t(r - B) + p_{1t}(A + B)] \\ &= b_t B + p_{2t}B + p_{1t}(-B) \\ &= B(b_t - \triangle_t) \\ &\geq 0. \end{aligned}$$

Thus action $\mathbf{b}''$ is better than action $\mathbf{b}$, contradicts with the assumption that action $\mathbf{b}$ is optimal. Therefore, it is always better to purchase only REC-bundled energy, i.e., $y_{1t}^* = y_{2t}^*$. Denote $w_t = \bar{x}_t + y_{1t}$, we can write the cost-to-go function as

$$\begin{aligned} & V_t(x_t, u_t, R_t, P_t) \\ = & \min_{\substack{y_{1t} \geq 0 \\ w_t}} \{b_t[(w_t - y_{1t}) - x_t] + p_{1t}y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\ = & - b_t x_t + \min_{\substack{y_{1t} \geq 0 \\ w_t}} \{(p_{1t} - b_t)y_{1t} + G_t(y_{1t}) + b_t w_t + \gamma E_t[V_{t+1}(w_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\}. \end{aligned}$$

Thus $y_{1t}^* = S_{1t}^L(p_{1t}, b_t)$.

(b) When $x_t \geq H_t(u_t, R_t, P_t)$, according to Proposition 1 it is optimal to sell RECs. Therefore, we can write the optimal action as type one $\mathbf{a} = (-r, A, B)$. By Lemma 3 we know that $r \geq A$. Consider action $\mathbf{a}'' = (-(r - A), 0, A + B)$. By taking either of action $\mathbf{a}$ or $\mathbf{a}''$, the utility gains $-r + A$ units of RECs and $A + B$ units of electricity. Compare the costs of these two actions, we have

$$\begin{aligned} cost(\mathbf{a}) - cost(\mathbf{a}'') &= -b_t r + p_{1t}A + p_{2t}B - [-b_t(r-A) + p_{2t}(A+B)] \\ &= -b_t r - p_{2t}A + p_{1t}A \\ &= A(\triangle_t - b_t) \\ &\geq 0. \end{aligned}$$

Thus action $\mathbf{a}''$ is better than action $\mathbf{a}$, contradicts with the assumption that action $\mathbf{a}$ is optimal. Therefore, it is always better to purchase only regular energy, i.e., $y_{1t}^* = 0$. We can write the cost-to-go function as

$$\begin{aligned} & V_t(x_t, u_t, R_t, P_t) \\ =& \min_{\substack{y_{2t} \geq 0 \\ \bar{x}_t}} \{-s_t(x_t - \bar{x}_t) + p_{2t}y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\ =& - s_t x_t + \min_{\substack{y_{2t} \geq 0 \\ \bar{x}_t}} \{p_{2t}y_{2t} + G_t(y_{2t}) + s_t\bar{x}_t + \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\}. \end{aligned}$$

Thus $y_{2t}^* = S_{2t}(p_{2t})$, $H_t(u_t, R_t, P_t) = \bar{x}_t^* = w_t^H(u_t, R_t, P_t)$.

Theorem 3. *In each period $t = 1, \ldots, T$, given state (x_t, u_t, R_t, P_t), if $s_t < \triangle_t < b_t$, there exists a pair of thresholds $(l_t(u_t, R_t, P_t), h_t(u_t, R_t, P_t))$ satisfying*

$$L_t(u_t, R_t, P_t) \leq l_t(u_t, R_t, P_t) \leq h_t(u_t, R_t, P_t) \leq H_t(u_t, R_t, P_t),$$

such that the utility's optimal purchasing quantities in the forward market are

(a) *When $L_t(u_t, R_t, P_t) < x_t \leq l_t(u_t, R_t, P_t)$, it is optimal to purchase only REC-bundled energy, and $y_{1t}^* = y_{2t}^* = s_{1t}(x_t, u_t, R_t, P_t)$;*
(b) *When $l_t(u_t, R_t, P_t) < x_t < h_t(u_t, R_t, P_t)$, it is optimal to purchase both REC-bundled energy and regular energy, and $y_{1t}^* = w_t^{\triangle}(u_t, R_t, P_t) - x_t$, $y_{2t}^* = S_{2t}(p_{2t})$;*
(c) *When $h_t(u_t, R_t, P_t) \leq x_t < H_t(u_t, R_t, P_t)$, it is optimal to purchase only regular energy, and $y_{1t}^* = 0$, $y_{2t}^* = S_{2t}(p_{2t})$.*

Proof of Theorem 3:
If $s_t < \triangle_t < b_t$, when $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$, $\bar{x}_t^* = x_t$ (Proposition 1). Denote $w_t = \bar{x}_t + y_{1t}$, we can write the cost-to-go function as

$$
\begin{aligned}
&V_t(x_t,u_t,R_t,P_t) = W_t(x_t,u_t,R_t,P_t) \\
&= \min_{\substack{y_{1t}\geq 0 \\ y_{2t}\geq y_{1t}}} \{p_{1t}y_{1t} + p_{2t}(y_{2t}-y_{1t}) + G_t(y_{2t}) + \gamma E_t[V_{t+1}(w_t, u_t+D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\
&= \min_{\substack{y_{1t}\geq 0 \\ y_{2t}\geq y_{1t}}} \{(p_{1t}-p_{2t})y_{1t} + p_{2t}y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(w_t, u_t+D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\
&= \min_{\substack{w_t\geq x_t \\ y_{2t}\geq w_t-x_t}} \{\triangle_t(w_t-x_t) + p_{2t}y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(w_t, u_t+D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\
&= \min_{\substack{w_t\geq x_t \\ y_{2t}\geq w_t-x_t}} \{-\triangle_t x_t + p_{2t}y_{2t} + G_t(y_{2t}) + \triangle_t w_t + \gamma E_t[V_{t+1}(w_t, u_t+D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1}]\} \\
&= \min_{\substack{w_t\geq x_t \\ y_{2t}\geq w_t-x_t}} f(x_t,u_t,R_t,P_t,w_t,y_{2t}).
\end{aligned} \tag{4}
$$

Denote $(w_t^*, y_{2t}^*) = \arg\min_{\substack{w_t\geq x_t \\ y_{2t}\geq w_t-x_t}} f(x_t,u_t,R_t,P_t,w_t,y_{2t})$ as the optimal solutions to this optimization problem. Note $f(x_t,u_t,R_t,P_t,w_t,y_{2t})$ is a separate convex function on (w_t, y_{2t}). Define

$$
w_t^{\triangle}(u_t,R_t,P_t) = \arg\min_{w_t}\{\triangle_t w_t + \gamma E_t[V_{t+1}(w_t, u_t+D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}
$$

as the utility's optimal REC level at the end of period t when the utility does not trade unbundled RECs. Then $(w_t^{\triangle}(u_t,R_t,P_t),\ S_{2t}(p_{2t}))$ is the global minimum of f on $\mathbf{R}^2$ plane. W.l.o.g, we assume this global optimum is unique.

As x_t increases from $-\infty$ to $+\infty$, the feasible region $\{(w_t, y_{2t}) \in \mathbf{R}^2 : w_t \geq x_t, y_{2t} \geq w_t - x_t\}$ moves towards the right hand side. In the following we divide the range of x_t $(L_t(u_t,R_t,P_t) < x_t < H_t(u_t,R_t,P_t))$ into three cases, so that we can discuss under each of these three cases, whether or not the feasible region contains the global minimum as an interior point.

To this end, define

$$
\begin{aligned}
l_t(u_t,\ R_t,P_t) =& w_t^{\triangle}(u_t,\ R_t,P_t) - S_{2t}(p_{2t}), \\
h_t(u_t,\ R_t,P_t) =& w_t^{\triangle}(u_t,\ R_t,P_t).
\end{aligned}
$$

First we show when $s_t < \triangle_t < b_t$, $L_t(u_t,R_t,P_t) \leq l_t(u_t,\ R_t,P_t) \leq h_t(u_t,\ R_t,P_t) \leq H_t(u_t,R_t,P_t)$, so that we can divide $L_t(u_t,R_t,P_t) < x_t < H_t(u_t,R_t,P_t)$ into three cases: $L_t(u_t,R_t,P_t) < x_t \leq l_t(u_t,\ R_t,P_t)$, $l_t(u_t,\ R_t,P_t) < x_t < h_t(u_t,\ R_t,P_t)$, and $h_t(u_t,R_t,P_t) \leq x_t < H_t(u_t,R_t,P_t)$.
From the definition of $l_t(u_t,\ R_t,P_t)$ and $h_t(u_t,\ R_t,P_t)$, we know that it is equivalent to show

$$
L_t(u_t,\ R_t,\ P_t) \leq w_t^{\triangle}(u_t,\ R_t,P_t) - S_{2t}(p_{2t}) \leq w_t^{\triangle}(u_t,\ R_t,P_t) \leq H_t(u_t,\ R_t,\ P_t).
$$

Let us start with the first inequality. When $s_t < \triangle_t < b_t$, we have $L_t(u_t,\ R_t,P_t) = w_t^L(u_t,\ R_t,P_t) - S_{1t}^L(p_{1t},b_t)$ (Theorem 1). Thus in order to prove the first inequality, it is sufficient to show $w_t^L(u_t,\ R_t,P_t) \leq w_t^{\triangle}(u_t,\ R_t,P_t)$ and $S_{1t}^L(p_{1t},b_t) \geq$

$S_{2t}(p_{2t})$. From the definition of $w_t^L(u_t,\, R_t, P_t)$, $w_t^{\triangle}(u_t,\, R_t, P_t)$, and the submodularity of $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ on (x_{t+1}, u_{t+1}), we know that $w_t^L(u_t,\, R_t, P_t) \leq w_t^{\triangle}(u_t,\, R_t, P_t)$. On the other hand, from the definition of $S_{1t}^L(p_{1t}, b_t)$, $S_{2t}(p_{2t})$ and the convexity of $G_t(y_{1t})$, we know that $S_{1t}^L(b_t, p_{1t}) \geq S_{2t}(p_{2t})$. Thus we have proved the first inequality.

The second inequality is straightforward since $S_{2t}(p_{2t}) \geq 0$.

Let us prove the third inequality. When $s_t < \triangle_t < b_t$, $H_t(u_t, R_t, P_t) = w_t^H(u_t, R_t, P_t)$ (Theorem 2). Thus the third inequality is equivalent as $w_t^{\triangle}(u_t, R_t, P_t) \leq w_t^H(u_t, R_t, P_t)$. From the definition of $w_t^{\triangle}(u_t, R_t, P_t)$, $w_t^H(u_t, R_t, P_t)$, and the submodularity of $V_{t+1}(x_{t+1}, u_{t+1}, R_{t+1}, P_{t+1})$ on (x_{t+1}, u_{t+1}), we know that $w_t^{\triangle}(u_t, R_t, P_t) \leq w_t^H(u_t, R_t, P_t)$. Thus the third inequality holds. We have proved $L_t(u_t, R_t, P_t) \leq l_t(u_t,\, R_t, P_t) \leq h_t(u_t,\, R_t, P_t) \leq H_t(u_t, R_t, P_t)$.

In the following, we discuss under each of these three cases whether or not the feasible region contains the global minimum as an interior point.

(a) When $L_t(u_t, R_t, P_t) < x_t \leq l_t(u_t,\, R_t, P_t)$, the feasible region is on the left-hand-side of the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\, S_{2t}(p_{2t}))$ and does not include it as an interior point. Since $f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ is jointly convex on (w_t, y_{2t}), the optimal solution to optimization problem (4), (w_t^*, y_{2t}^*), is on the right boundary of the feasible region. Thus $y_{2t}^* = w_t^* - x_t$. Therefore, $y_{2t}^* = y_{1t}^*$, the utility should purchase only REC-bundled energy. We have

$$
\begin{aligned}
&V_t(x_t, u_t, R_t, P_t) = W_t(x_t, u_t, R_t, P_t) \\
&= \min_{y_{1t} \geq 0} \{p_{1t} y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(x_t + y_{1t}, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}.
\end{aligned}
$$

Thus $y_{1t}^* = y_{2t}^* = s_{1t}(x_t, u_t, R_t, P_t)$.

(b) When $l_t(u_t,\, R_t, P_t) < x_t < h_t(u_t,\, R_t, P_t)$, the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\, S_{2t}(p_{2t}))$ is in the interior of the feasible region. Thus the global minimum is the optimal solution to optimization problem (4). In this case, the utility should purchase both REC-bundled energy and regular energy. We have $(w_t^*, y_{2t}^*) = (w_t^{\triangle}(u_t, R_t, P_t),\, S_{2t}(p_{2t}))$.
Thus $y_{1t}^* = w_t^{\triangle}(u_t, R_t, P_t) - x_t$, $y_{2t}^* = S_{2t}(p_{2t})$.

(c) When $h_t(u_t, R_t, P_t) \leq x_t < H_t(u_t, R_t, P_t)$, the feasible region is on the right-hand-side of the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\, S_{2t}(p_{2t}))$ and does not include it as an interior point. Since $f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ is jointly convex on (w_t, y_{2t}), the optimal solution to optimization problem (4), (w_t^*, y_{2t}^*), is on the left boundary of the feasible region. Thus $w_t^* = x_t$. Therefore, $y_{1t}^* = 0$, and the utility should purchase only regular energy. We have

$$
\begin{aligned}
&V_t(x_t, u_t, R_t, P_t) \\
&= \min_{y_{2t} \geq 0, \bar{x}_t} \{p_{2t} y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, \tilde{R}_{t+1}, \tilde{P}_{t+1})]\}.
\end{aligned}
$$

Thus $y_{1t}^* = 0$, $y_{2t}^* = S_{2t}(p_{2t})$.

References

Boyd, S. P., Vandenberghe, L., 2004. Convex optimization. Cambridge University Press.

Topkis, D. M., 1998. Supermodularity and complementarity. Princeton University Press.